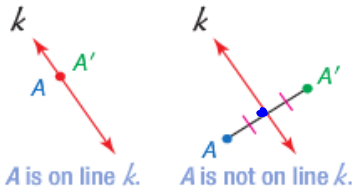
1 Draw Reflections In Lesson 4-7, you learned that a reflection or *flip* is a transformation in a line called the **line of reflection**. Each point of the preimage and its corresponding point on the image are the same distance from this line.

KeyConcept Reflection in a Line

A reflection in a line is a function that maps a point to its image such that

- if the point is on the line, then the image and preimage are the same point, or
- if the point is not on the line, the line is the perpendicular bisector of the segment joining the two points.



A′, *A″*, *A‴*, and so on, name corresponding points for one or more transformations.

To reflect a polygon in a line, reflect each of the polygon's vertices. Then connect these vertices to form the reflected image.

Example 1 Reflect a Figure in a Line

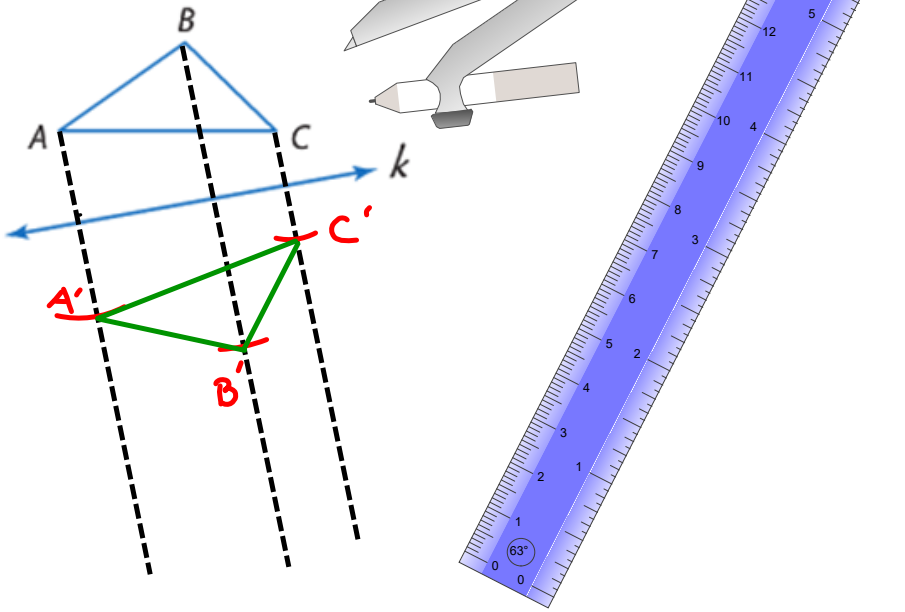
Copy the figure and the given line of reflection. Then draw the reflected image in this line using a ruler.

Step 1 Draw a line through each vertex that is perpendicular to line k.

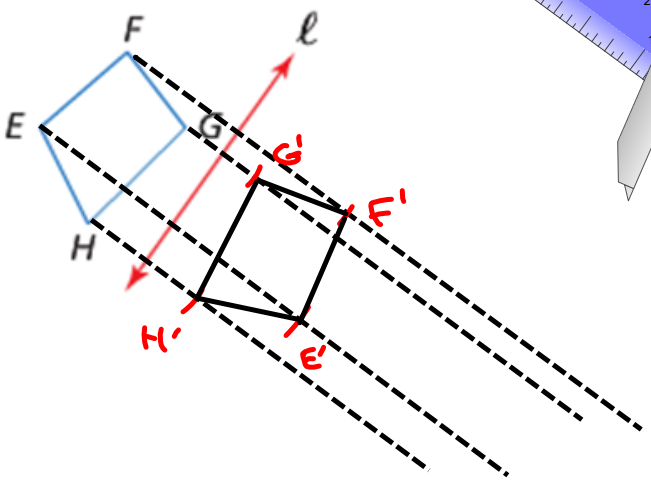
Step 2 Measure the distance from point A to line k. Then locate A' the same distance from line k on the opposite side

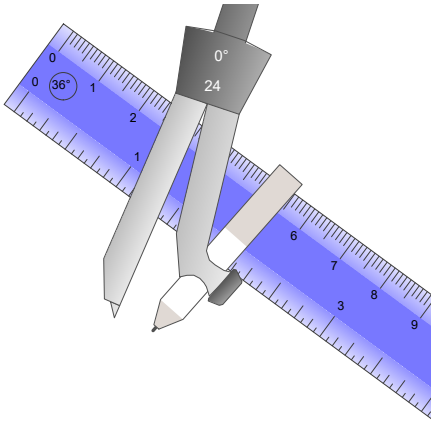
Step 3 Repeat Step 2 to locate points B' and C'. Then connect vertices A', B', and C' to form the reflected image.

$\triangle A'B'C'$ is a reflection of $\triangle ABC$ over line k

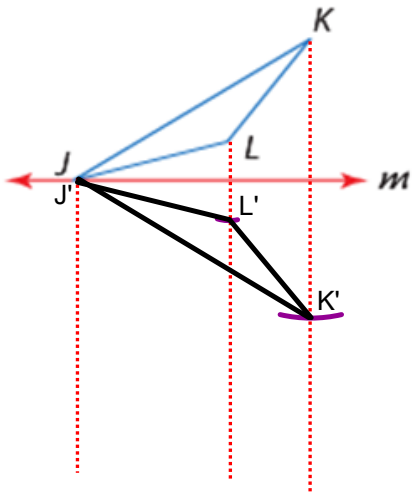


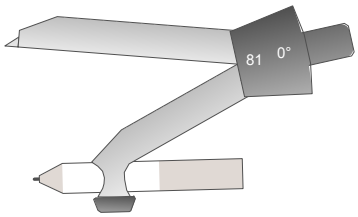
1A.

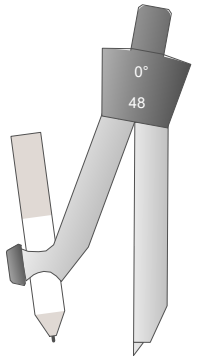




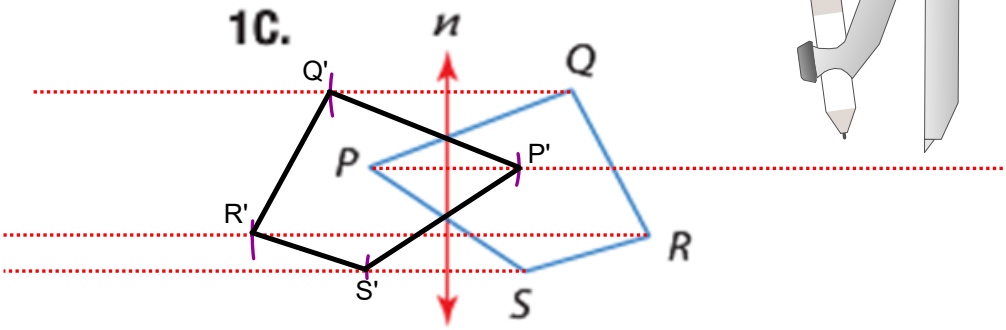
1B.







1C.

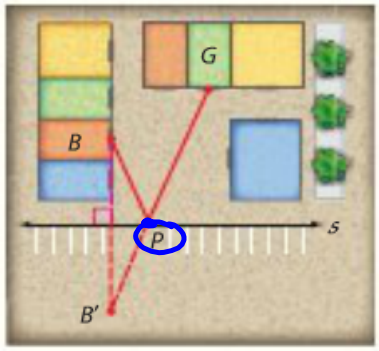


Real-World Example 2 Minimize Distance by Using a Reflection

SHOPPING **Suppose you are going to buy clothes in Store B, return to your car, and then buy shoes at Store G. Where along line s of parking spaces should you park to minimize the distance you will walk?**

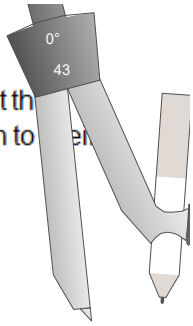
Understand You are asked to locate a point P on line s such that $BP + PG$ has the least possible value.

Plan The total distance from B to P and then from P to G is least when these three points are collinear. Use the reflection of point B in line s to find the location for point P.



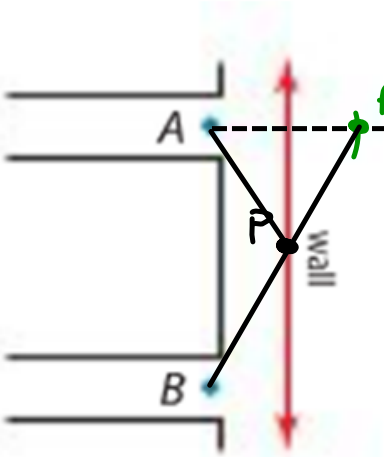
Solve Draw $\overline{B'G}$. Locate P at the intersection of line s and $\overline{B'G}$.

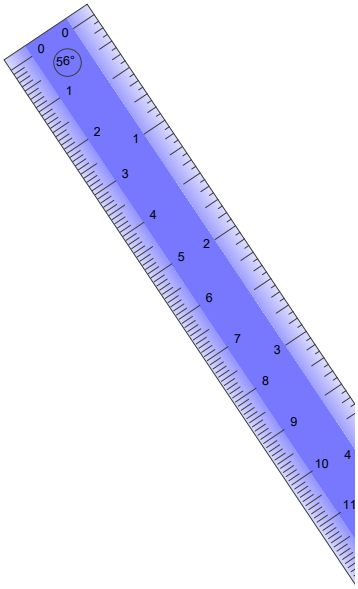
Check Compare the sum $BP + PG$ for each case to verify that the location found for P minimizes this sum.



ex. 2 Joy wants to select a good location to sell tickets for a dance. Locate point P such that th distance someone would have to walk from Hallway A, to point P on the wall, and then to e next class in Hallway B is minimized.

① find A'
② connect A' with B to form $\overline{A'B}$.
③ label the intersection of A'B with the wall as P.
④ connect A with P.





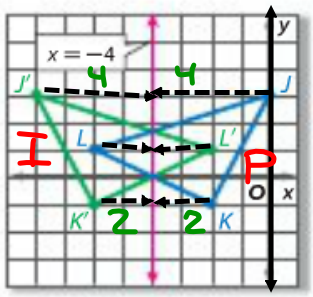
Example 3 Reflect a Figure in a Horizontal or Vertical Line

Triangle JKL has vertices $J(0, 3)$, $K(-2, -1)$, and $L(-6, 1)$. Graph $\triangle JKL$ and its image in the given line.

line of reflection

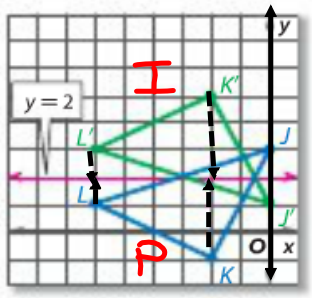
a. $x = -4$

Find a corresponding point for each vertex so that a vertex and its image are equidistant from the line $x = -4$.

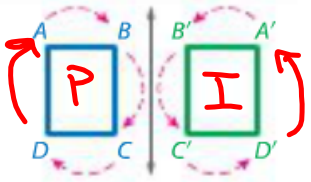


b. $y = 2$

Find a corresponding point for each vertex so that a vertex and its image are equidistant from the line $y = 2$.



Characteristics of a Reflection Reflections, like all isometries, preserve distance, angle measure, betweenness of points, and collinearity. The orientation of a preimage and its image, however, are reversed.



cw ccw

What type of isometry is a reflection?

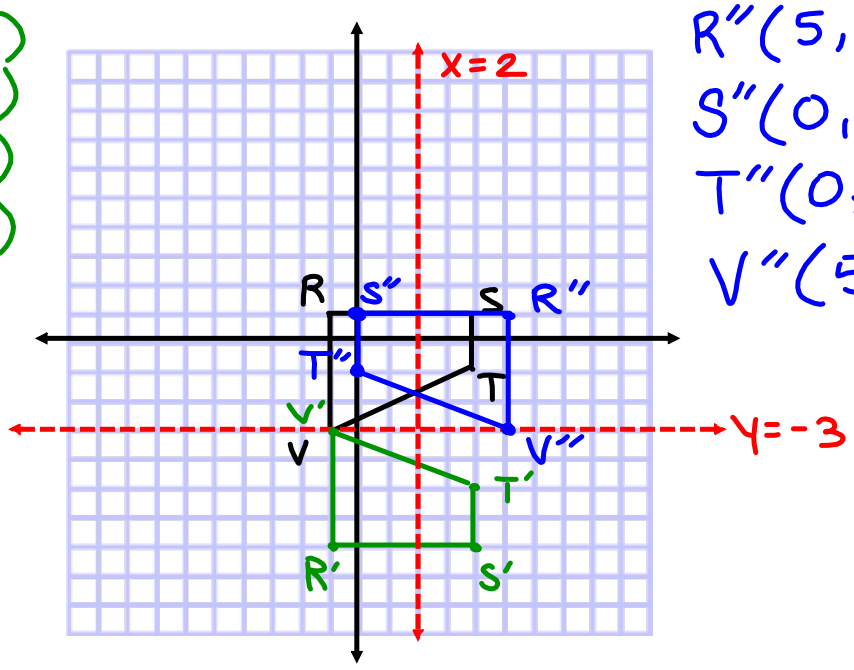
indirect isometry

ex. 3

Trapezoid RSTV has vertices R(-1, 1), S(4, 1), T(4, -1), and V(-1, -3). Graph trapezoid RSTV and its image in the given line.

3A. y = -3 3B. x = 2

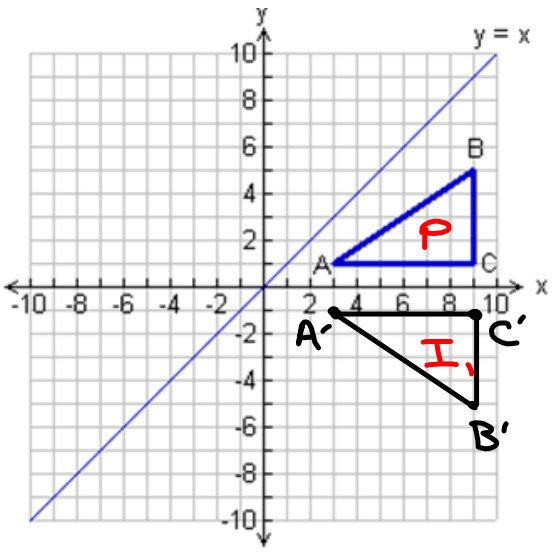
R′(-1,-7)
S′(4,-7)
T′(4,-5)
V′(-1,-3)



R″(5,1)
S″(0,1)
T″(0,-1)
V″(5,-3)

Reflections in the x-axis, y-axis, and the line y = x

Graph the image of △ABC after the transformation $r_{x\text{-axis}}$. Label and state the coordinates.



reflection over the x-axis

notation: $r_{x\text{-axis}}$

reflection → P

subscript → I

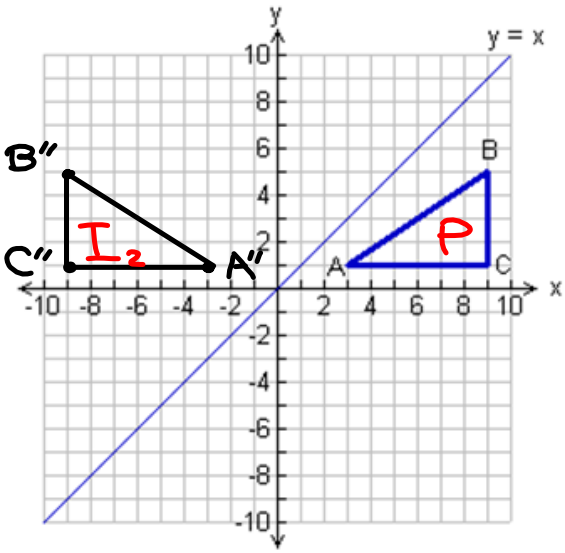
$A(3,1) \rightarrow A'(3,-1)$

$B(9,5) \rightarrow B'(9,-5)$

$C(9,1) \rightarrow C'(9,-1)$

$P(x,y) \rightarrow I(x,-y)$

Graph the image of △ABC after the transformation $r_{y\text{-axis}}$. Label and state the coordinates.



$r_{y\text{-axis}}$

reflection over the y-axis

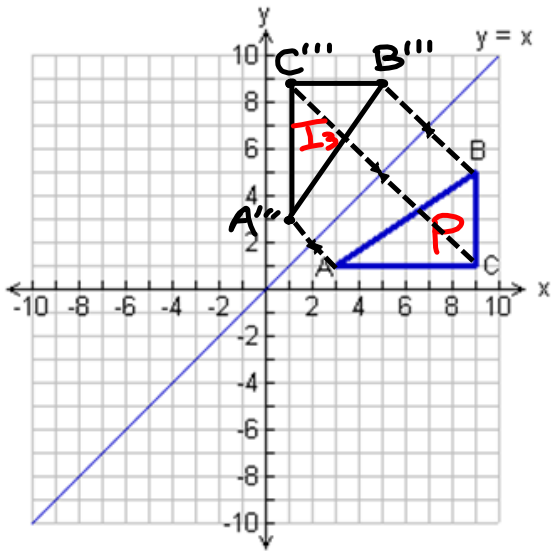
$A(3,1) \longrightarrow A''(-3,1)$

$B(9,5) \longrightarrow B''(-9,5)$

$C(9,1) \longrightarrow C''(-9,1)$

$P(x,y) \longrightarrow I(-x,y)$

Graph the image of △ABC after the transformation $r_{y=x}$. Label and state the coordinates.



$r_{y=x}$

reflection over $y=x$

$A(3,1) \longrightarrow A'''(1,3)$

$B(9,5) \longrightarrow B'''(5,9)$

$C(9,1) \longrightarrow C'''(1,9)$

$P(x,y) \longrightarrow I(y,x)$

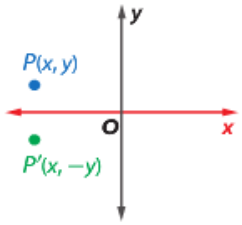
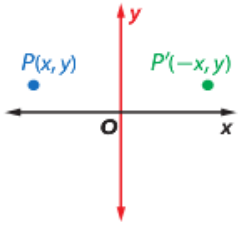
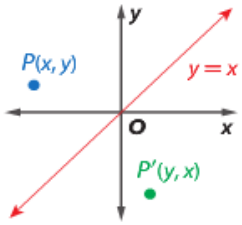
Based on the graph above, determine the **Reflection Coordinate Rules**?

Reflection over the x-axis ($r_{x\text{-axis}}$):
$(x, y) \rightarrow$ (x, -y)

Reflection over the y-axis ($r_{y\text{-axis}}$):
$(x, y) \rightarrow$ (-x, y)

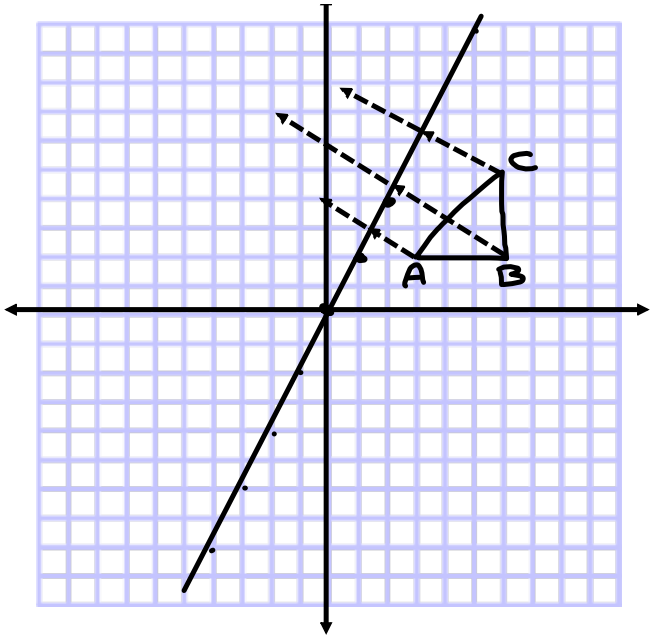
Reflection over the line $y = x$ ($r_{y=x}$):
$(x, y) \rightarrow$ (y, x)

ConceptSummary Reflection in the Coordinate Plane

Reflection in the x-axis	Reflection in the y-axis	Reflection in the line $y = x$
		
$(x, y) \rightarrow (x, -y)$	$(x, y) \rightarrow (-x, y)$	$(x, y) \rightarrow (y, x)$

$y = 2x$

x	y
0	0
1	2
2	4
3	6
4	8
5	10



A = (3,2)

B (6,2)

C (6,5)